Compton polarimetry by scattered electrons (translated to english)

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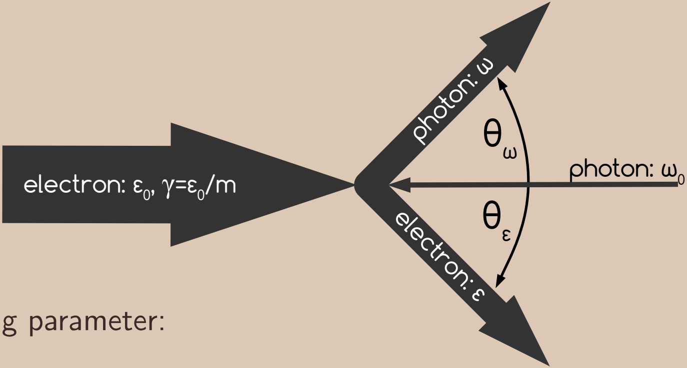
Inverse Compton scattering: kinematics

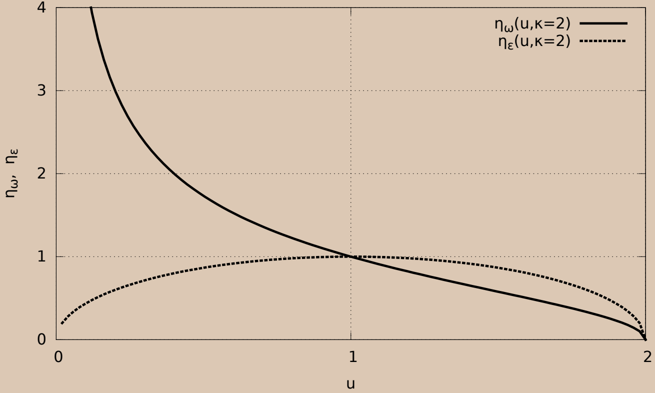


Scattering parameter:

$$u = \frac{\omega}{\varepsilon} = \frac{\theta_\varepsilon}{\theta_\omega} = \frac{\omega}{\varepsilon_0 - \omega} = \frac{\varepsilon_0 - \varepsilon}{\varepsilon}; \quad u \in [0, \kappa]\text{, where } \kappa = \frac{4\omega_0\varepsilon_0}{m^2}.$$

Scattering angles: $\eta_\omega \equiv \gamma\theta_\omega = \sqrt{\frac{\kappa}{u} - 1}; \quad \eta_\varepsilon \equiv \gamma\theta_\varepsilon = u\sqrt{\frac{\kappa}{u} - 1}.$

Inverse Compton scattering: kinematics



$\kappa = 2$ when, e. g., $\omega_0 = 2.33$ eV and $\varepsilon_0 = 56$ GeV.

For any κ, if $u = \dfrac{\kappa}{2}$ one has $\eta_\omega = 1$, $\eta_\varepsilon = \dfrac{\kappa}{2}$, i. e. $\max(\theta_\varepsilon) = \dfrac{2\omega_0}{m}$.
Maximum electron scattering angle does not depend on beam energy!

Inverse Compton scattering: cross section

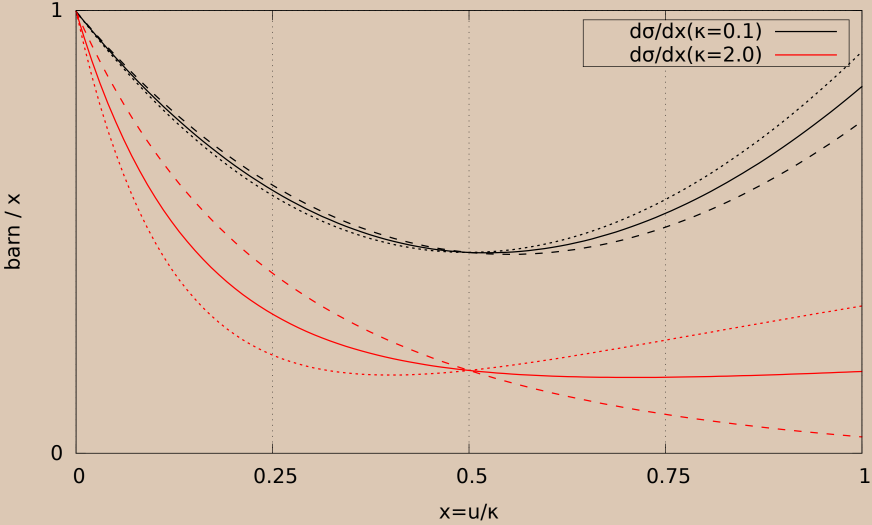
$$d\sigma = \left\{ \frac{1}{\kappa(1+u)^2} \left(2 + \frac{u^2}{1+u} + 4\frac{u}{\kappa}\left[\frac{u}{\kappa} - 1\right]\left[1 - \xi_\perp \cos(2(\varphi - \varphi_\perp))\right] \right) + \right.$$

$$\left. + \, \xi_\circlearrowright \left(\zeta_\parallel \frac{u(u+2)(\kappa - 2u)}{\kappa^2(1+u)^3} - \zeta_\perp \frac{2u^2\sqrt{\kappa/u - 1}}{\kappa^2(1+u)^3} \sin\varphi \right) \right\} r_e^2 d\varphi\, du,$$

Modified Stokes parameters:

$\xi_\perp$ и $\varphi_\perp$ – degree and direction of laser linear polarization,
$\xi_\circlearrowright$ – degree of laser circular polarization,
$\zeta_\parallel$ – longitudinal electron beam polarization degree,
$\zeta_\perp$ – transverse vertical electron beam polarization degree.
Laser beam: $\sqrt{\xi_\perp^2 + \xi_\circlearrowright^2} = 1$, $\xi_\perp \in [0,1]$, $\xi_\circlearrowright \in [-1,1]$.
Electron beam: $\sqrt{\zeta_\perp^2 + \zeta_\parallel^2} < 1$, $\zeta_\perp \in [-1,1]$, $\zeta_\parallel \in [-1,1]$.

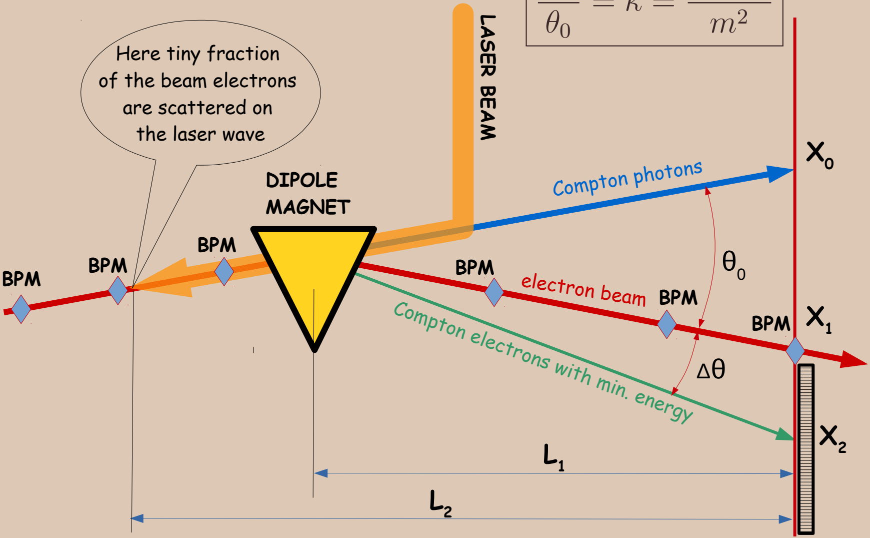
Inverse Compton scattering: cross section



Dashed lines illustrate the influence of $\xi_{\circlearrowright}\zeta_{\parallel}$

General concept of I.C.S. experiments

$$\frac{\Delta\theta}{\theta_0} = \kappa = \frac{4\omega_0 E_0}{m^2}$$



Compton polarimeters for electron beam

VEPP-2, VEPP-4, LEP, HERA, SLC ... ILC, FCC ...

At first Compton polarimeters it was general to deal with backscattered photons.

At higher electron energies the divergence of γ-beam is small, high energy SR photons appear, etc., it seams reasonable to look at scattered electrons.

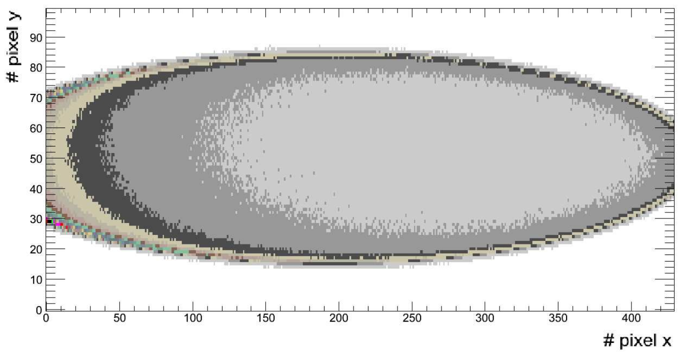
Maximum scattering angle of electron does not depend on its initial energy: $\max(\theta_\varepsilon) = 2\omega_0/m$.
If ω_0 =2.33 eV one has $\max(\theta_\varepsilon) \simeq 10$ μrad.
Vertical beam energy spread: $\sigma'_y = \sqrt{\epsilon_y/\beta_y} \ll \max(\theta_\varepsilon)$.
An example: ϵ_y=100 pm и β_y=100 m gives $\sigma'_y = 1$ μrad.

ILC note: LC-M-2012-001

A Transverse Polarimeter for a Linear Collider of 250 GeV e Beam Energy

Itai Ben Mordechai and Gideon Alexander

"... For the detection of the scattered electrons we consider only a position measurement using a Silicon pixel detector placed at a distance of 37.95 m from the Compton IP. The active dimension of the detector is 2×200 mm^2. The size of the pixels cell taken is 50×400 μm^2 similar to the one used in the ATLAS detector [9]. This scheme yields an approximate two dimensional resolution of 14.4×115.5 μm^2 [10] with a data read-out rate of ..."



Angles of scattered electron after bending dipole

The energy of scattered electron depends on u as: $\varepsilon = \varepsilon_0/(1+u)$. Such an electron will be bent in a dipole to an angle:

$$\theta_s = \frac{A}{\varepsilon} = \frac{A}{\gamma m}(1+u); \qquad \eta_s \equiv \gamma\theta_s = \eta_0 + u\eta_0,$$

where A is the dipole "force", $\gamma = \varepsilon_0/m$, $\eta_0 = A/m$.

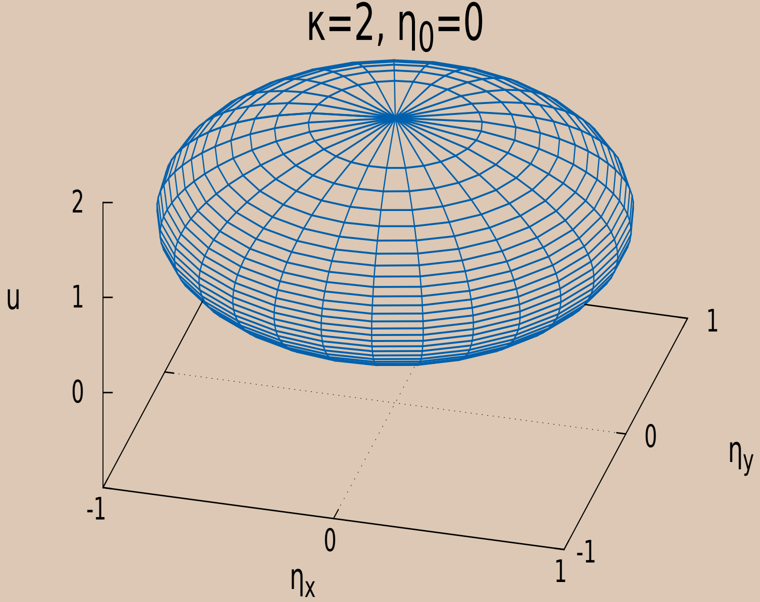
Introducing new parameters η_x, η_y:

$$\begin{cases} \eta_x \equiv \eta_s - \eta_0 = u\eta_0 + & u\sqrt{\kappa/u - 1}\cos\varphi \\ \eta_y = & u\sqrt{\kappa/u - 1}\sin\varphi \end{cases}$$

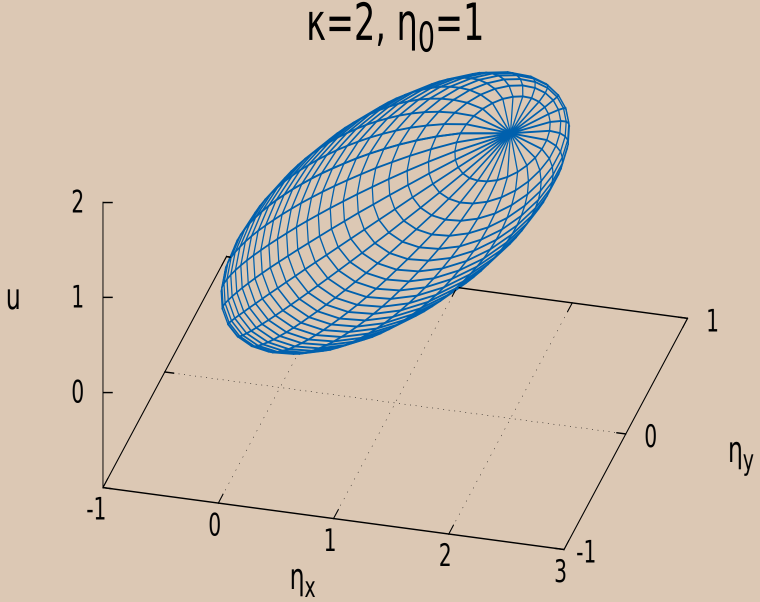
one gets the square equation for u:

$$(\eta_x - u\eta_0)^2 + \eta_y^2 = u(\kappa - u).$$

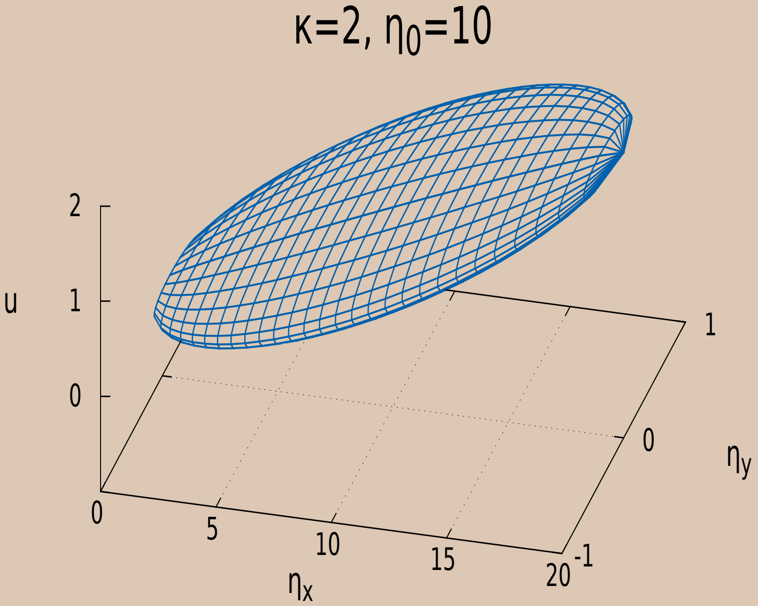
Scattering surface



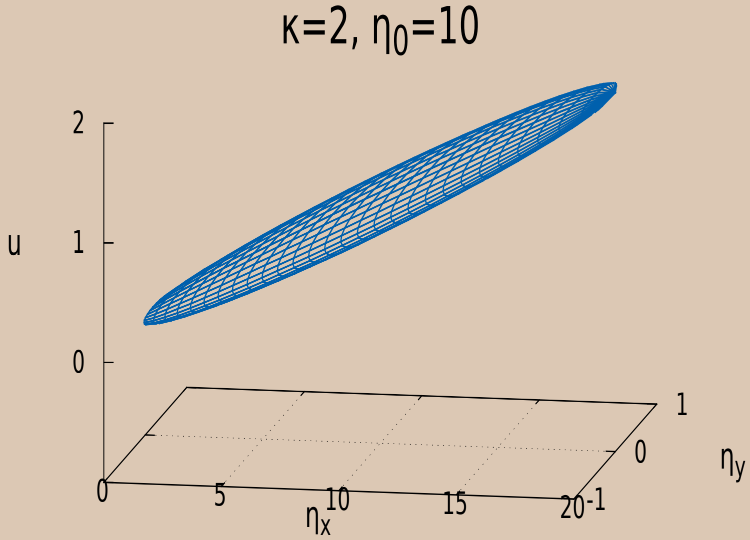
Scattering surface



Scattering surface

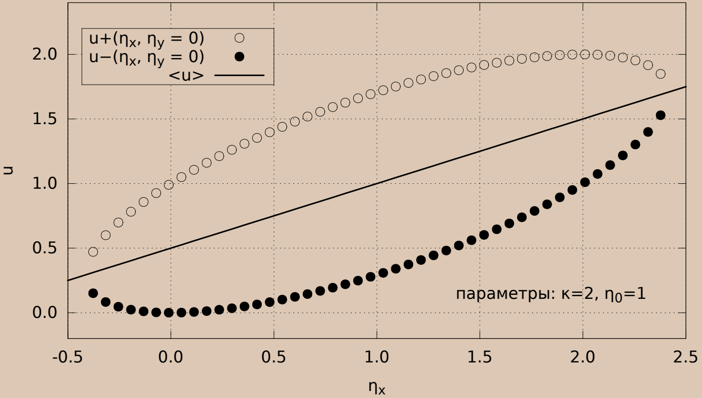


Scattering surface



Solving the equation:

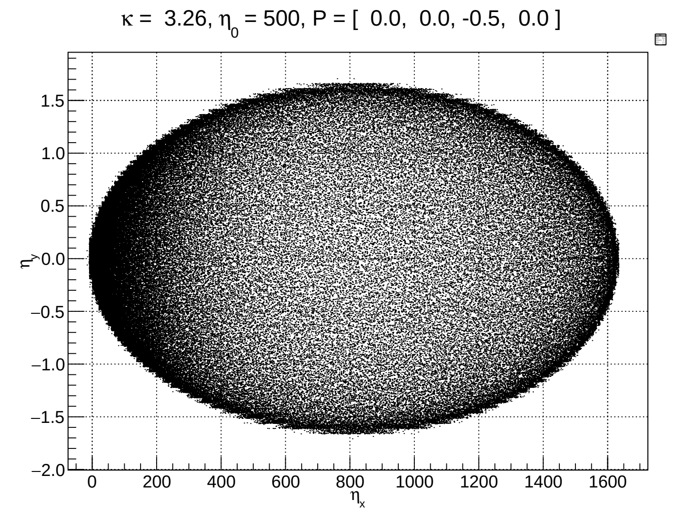
$$u = \frac{\kappa/2 + \eta_0\eta_x \pm 2J}{1+\eta_0^2}, \text{ where } J = \sqrt{\kappa^2/4 - \eta_x^2 - \eta_y^2(1+\eta_0^2) + \kappa\eta_0\eta_x}$$



$$\langle u\rangle = \frac{\kappa/2 + \eta_0\eta_x}{1+\eta_0^2} \xrightarrow{\eta_0 \gg 1} \frac{\eta_x}{\eta_0}.$$

And also: $d\varphi du = d\eta_x d\eta_y / J$

Ellipse in η_x, η_y plane



Sizes of ellipse are [rad]: $O_y = \frac{4\omega_0}{m}$, $O_x = \frac{4\omega_0}{m}\sqrt{1 + (\gamma\theta_0)^2}$

Cross section: $(u, \varphi) \to (\eta_x, \eta_y)$

$$d\sigma = \left\{ \frac{1}{\kappa(1+u)^2} \left(2 + \frac{u^2}{1+u} + 4\frac{u}{\kappa}\left[\frac{u}{\kappa} - 1\right]\left[1 - \xi_\perp \cos(2(\varphi - \varphi_\perp))\right] \right) + \right.$$

$$\left. + \xi_\circlearrowright \left(\zeta_\parallel \frac{u(u+2)(\kappa - 2u)}{\kappa^2(1+u)^3} - \zeta_\perp \frac{2u^2\sqrt{\kappa/u - 1}}{\kappa^2(1+u)^3} \sin\varphi \right) \right\} r_e^2 d\varphi \, du =$$

$$= \left\{ \frac{1}{\kappa(1+u)^2} \left(2 + \frac{u^2}{1+u} + 4\frac{u}{\kappa}\left[\frac{u}{\kappa} - 1\right]\left[1 - \xi_\perp (1 - \frac{2\eta_y^2 \cos^2\varphi_\perp}{u(k-u)})\right] \right) + \right.$$

$$\left. + \xi_\circlearrowright \left(\zeta_\parallel \frac{u(u+2)(\kappa - 2u)}{\kappa^2(1+u)^3} - \zeta_\perp \frac{2u\eta_y}{\kappa^2(1+u)^3} \right) \right\} \frac{r_e^2 d\eta_x d\eta_y}{J(\eta_x, \eta_y)} \times 2.$$

Cross section: $(\eta_x, \eta_y) \to (r, \phi)$

$$d\sigma = \frac{\Sigma(\eta_x, \eta_y) d\eta_x d\eta_y}{\sqrt{\kappa^2/4 - \eta_x^2 - \eta_y^2(1+\eta_0^2) + \kappa\eta_0\eta_x}}$$

Ellipse→circle: $\left(x = \frac{\eta_x - \kappa\eta_0/2}{\sqrt{1+\eta_0^2}};\ y = \eta_y\right)$, then:

$$\begin{aligned}
J(x,y) = &\ \sqrt{(1+\eta_0^2)(\kappa^2/4 - x^2 - y^2)} = &\ \sqrt{(1+\eta_0^2)(\kappa^2/4 - r^2)}, \\
d\eta_x d\eta_y = &\ \sqrt{1+\eta_0^2}\, dx\, dy = &\ \sqrt{1+\eta_0^2}\, r\, dr\, d\phi, \\
\langle u \rangle = &\ \frac{\kappa}{2} + \frac{x\eta_0}{\sqrt{1+\eta_0^2}} = &\ \frac{\kappa}{2} + \frac{r\cos\phi \cdot \eta_0}{\sqrt{1+\eta_0^2}}.
\end{aligned}$$

Finally: $\boxed{d\sigma = \frac{\Sigma(r,\phi) r\, dr\, d\phi}{\sqrt{\kappa^2/4 - r^2}}}$

Take emittance into account: $f(\eta_x, \eta_y)$=

$$= \int\limits_0^R \frac{r\,dr}{\sqrt{R^2 - r^2}} \int\limits_0^{2\pi} \Sigma(r,\phi) \exp\left[-\frac{(r\cos\phi - \eta_x^*)^2}{2\sigma_x^{*2}} - \frac{(r\sin\phi - \eta_y)^2}{2\sigma_y^2}\right] d\phi,$$

where $R = \kappa/2, \quad \eta_x^* = (\eta_x - \kappa\eta_0/2)/\sqrt{1+\eta_0^2}, \quad \sigma_x^* = \sigma_x/\sqrt{1+\eta_0^2}$.

We divide the interval $[0 : R]$ intp N identical segments:

$$r_{i,B} = R \cdot i/N \equiv RB_i, \qquad r_{i,E} = R(i+1)/N \equiv RE_i.$$

$$\int\limits_{r_{i,B}}^{r_{i,E}} \frac{rdr}{\sqrt{R^2 - r^2}} = R\mathcal{I}_i; \quad \int\limits_{r_{i,B}}^{r_{i,E}} \frac{r^2dr}{\sqrt{R^2 - r^2}} = R^2\mathcal{J}_i$$

Weighted average radius of i ring: $\bar{r}_i = R \cdot \mathcal{J}_i/\mathcal{I}_i$.

The sum instead of integral: $f(\eta_x, \eta_y) =$

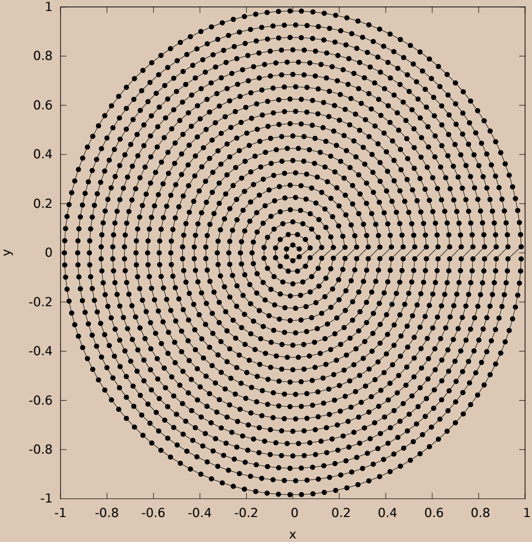
$$= R \sum_{i=0}^{N-1} \mathcal{I}_i \Delta\phi_i \sum_{j=0}^{\lfloor 2\pi(i+1) \rfloor} \Sigma_{ij} \exp\left[-\frac{\left(Rx_{ij} - \eta_x^*\right)^2}{2\sigma_x^{*2}} - \frac{\left(Ry_{ij} - \eta_y\right)^2}{2\sigma_y^2} \right],$$

where

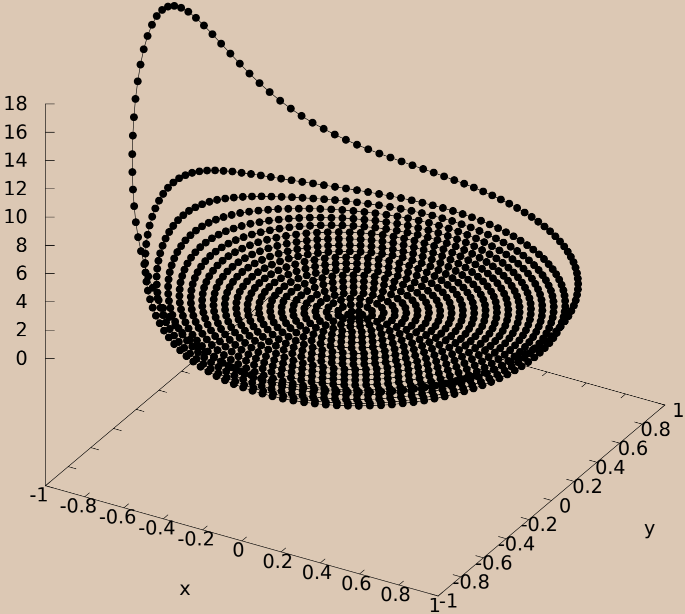
$$\begin{aligned}
\Delta\phi_i &= 2\pi / \lfloor 2\pi(i+1) \rfloor \\
x_{ij} &= \frac{\mathcal{J}_i}{\mathcal{I}_i} \cdot \frac{\sin((j+1)\Delta\phi_i) - \sin(j\Delta\phi_i)}{\Delta\phi_i} \\
y_{ij} &= \frac{\mathcal{J}_i}{\mathcal{I}_i} \cdot \frac{\cos(j\Delta\phi_i) - \cos((j+1)\Delta\phi_i)}{\Delta\phi_i} \\
u_{ij} &= R(1 + x_{ij}) \in [0 : 2R]
\end{aligned}$$

$$\Sigma_{ij} = \frac{(1 + x_{ij}^2)(1 + u_{ij}) + u_{ij}^2}{\kappa(1 + u_{ij})^3} - P_\parallel \frac{x_{ij} u_{ij}(2 + u_{ij})}{\kappa(1 + u_{ij})^3} - P_\perp \frac{y_{ij}(1 + x_{ij})}{2(1 + u_{ij})^3}$$

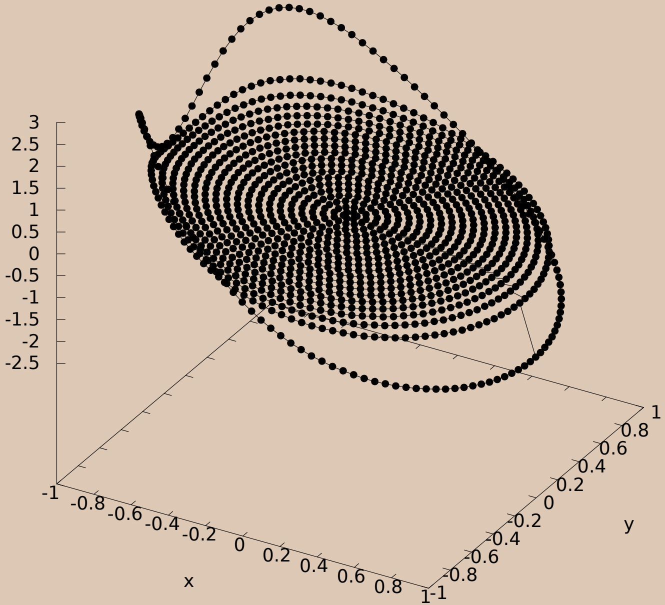
x_{ij} vs y_{ij}



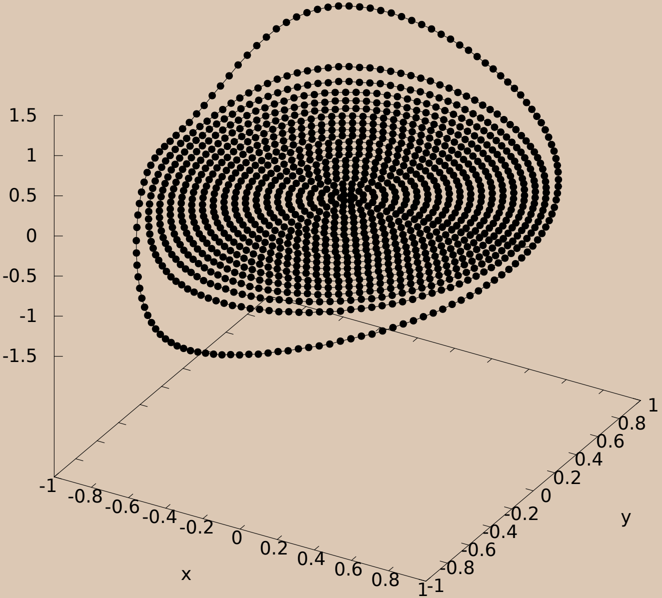
Unpolarized cross section



Contribution of 100% longitudinal polarization



Contribution of 100% transverse polarization



The function to fit the η_x, η_y distribution of scattered electrons:

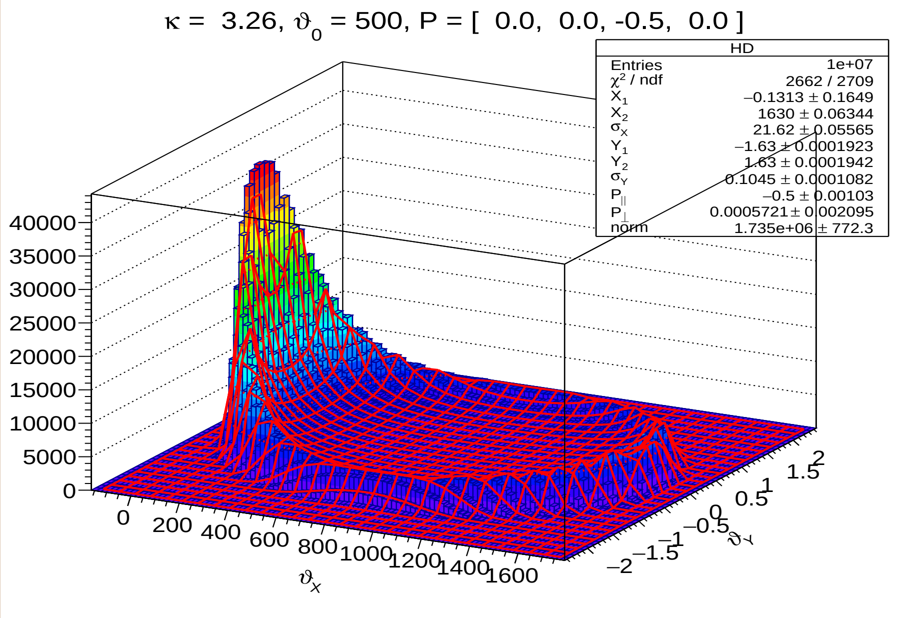
$$f(X,Y) = I \cdot \sum_{i,j} \frac{\mathcal{I}_i \Delta\varphi_i \Sigma_{ij}(P_\parallel, P_\perp)}{2\pi\sigma_X\sigma_Y} \times$$

$$\times \exp\left[-\frac{\left(\frac{X_2 - X_1}{2}\bar{x}_{ij} + \frac{X_1 + X_2}{2} - X\right)^2}{2\sigma_X^2}\right] \times$$

$$\times \exp\left[-\frac{\left(\frac{Y_2 - Y_1}{2}\bar{y}_{ij} + \frac{Y_1 + Y_2}{2} - Y\right)^2}{2\sigma_Y^2}\right].$$

Fit parameters: X_1, X_2, σ_X, Y_1, Y_2, σ_Y, $P_\parallel$, $P_\perp$, I (9 at all)

Fit results: longitudinal polarization 50%



Fit results: longitudinal polarization 50%

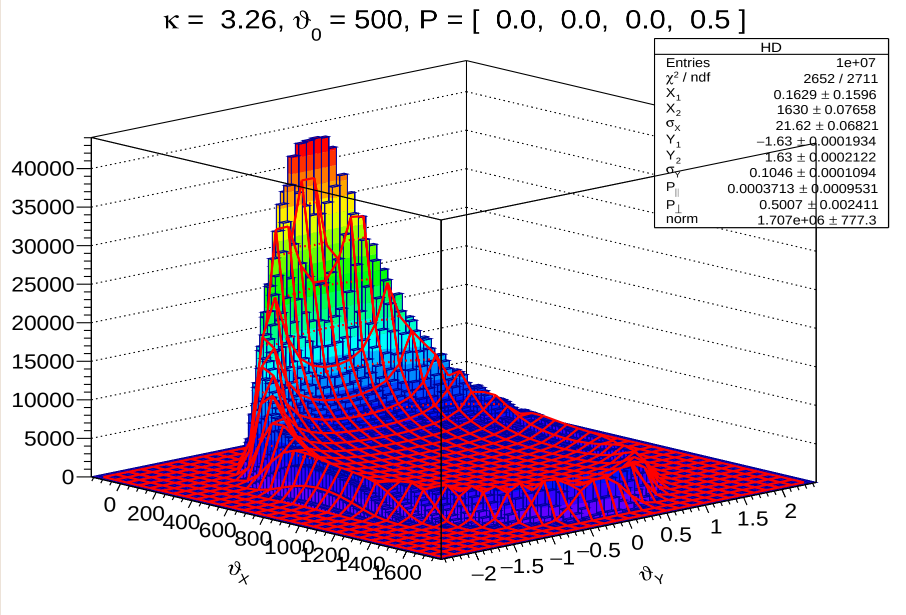
Fitting range along X – from 200

```
PARAMETER  CORRELATION COEFFICIENTS
 NO.  GLOBAL      1      2      3      4      5      6      7      8      9
  1  0.63806   1.000 -0.311  0.114 -0.332  0.339 -0.009 -0.360  0.018 -0.465
  2  0.61959  -0.311  1.000 -0.541  0.269 -0.282  0.123  0.219 -0.013  0.145
  3  0.57955   0.114 -0.541  1.000 -0.193  0.188 -0.262 -0.112  0.004 -0.052
  4  0.57920  -0.332  0.269 -0.193  1.000 -0.468  0.418  0.146  0.010  0.144
  5  0.60208   0.339 -0.282  0.188 -0.468  1.000 -0.443 -0.146  0.024 -0.166
  6  0.57481  -0.009  0.123 -0.262  0.418 -0.443  1.000  0.013 -0.005  0.022
  7  0.38479  -0.360  0.219 -0.112  0.146 -0.146  0.013  1.000 -0.003  0.231
  8  0.03962   0.018 -0.013  0.004  0.010  0.024 -0.005 -0.003  1.000 -0.003
  9  0.47128  -0.465  0.145 -0.052  0.144 -0.166  0.022  0.231 -0.003  1.000

 FCN=2662.5 FROM MIGRAD    STATUS=CONVERGED     257 CALLS         258 TOTAL
 EDM=3.93455e-08    STRATEGY= 1  ERROR MATRIX UNCERTAINTY   0.8 per cent
 EXT PARAMETER                  PARABOLIC         MINOS ERRORS
 NO.   NAME       VALUE            ERROR       NEGATIVE      POSITIVE
 1   X_{1}       -1.31308e-01   1.64882e-01
 2   X_{2}        1.62998e+03   6.34381e-02
 3   #sigma_{X}   2.16201e+01   5.56481e-02
 4   Y_{1}       -1.62981e+00   1.92272e-04
 5   Y_{2}        1.62973e+00   1.94174e-04
 6   #sigma_{Y}   1.04485e-01   1.08179e-04
 7   P_{#paral}  -5.00034e-01   1.02951e-03
 8   P_{#perp}    5.72060e-04   2.09542e-03
 9   norm         1.73486e+06   7.72345e+02
```

2DFit : Real Time = 140.38 seconds Cpu Time = 156.49 seconds

Fit results: transverse polarization 50%



Fit results: transverse polarization 50%

Fitting range along X – from 200

```
PARAMETER  CORRELATION COEFFICIENTS
NO.  GLOBAL      1      2      3      4      5      6      7      8      9
 1  0.69373   1.000 -0.318  0.138 -0.415  0.375 -0.015 -0.320  0.013 -0.537
 2  0.62430  -0.318  1.000 -0.556  0.270 -0.250  0.125  0.232 -0.003  0.154
 3  0.59633   0.138 -0.556  1.000 -0.185  0.189 -0.274 -0.145  0.012 -0.071
 4  0.62861  -0.415  0.270 -0.185  1.000 -0.432  0.432  0.174  0.095  0.206
 5  0.58723   0.375 -0.250  0.189 -0.432  1.000 -0.397 -0.137  0.153 -0.210
 6  0.58308  -0.015  0.125 -0.274  0.432 -0.397  1.000  0.022 -0.008  0.017
 7  0.36220  -0.320  0.232 -0.145  0.174 -0.137  0.022  1.000  0.011  0.243
 8  0.23947   0.013 -0.003  0.012  0.095  0.153 -0.008  0.011  1.000  0.022
 9  0.54496  -0.537  0.154 -0.071  0.206 -0.210  0.017  0.243  0.022  1.000

FCN=2651.75 FROM MIGRAD    STATUS=CONVERGED     258 CALLS         259 TOTAL
EDM=4.09631e-07    STRATEGY= 1  ERROR MATRIX UNCERTAINTY   0.4 per cent
EXT PARAMETER                  PARABOLIC         MINOS ERRORS
NO.   NAME      VALUE            ERROR      NEGATIVE      POSITIVE
 1  X_{1}        1.62941e-01   1.59586e-01
 2  X_{2}        1.63002e+03   7.65815e-02
 3  #sigma_{X}   2.16220e+01   6.82096e-02
 4  Y_{1}       -1.62982e+00   1.93423e-04
 5  Y_{2}        1.63003e+00   2.12161e-04
 6  #sigma_{Y}   1.04595e-01   1.09394e-04
 7  P_{#paral}   3.71312e-04   9.53123e-04
 8  P_{#perp}    5.00724e-01   2.41133e-03
 9  norm         1.70728e+06   7.77293e+02
```

2DFit : Real Time = 131.11 seconds Cpu Time = 158.11 seconds

Results & discussion

- Analysis of scattered electrons (SE) properties was performed.
- $X - Y$ distribution of SE after a bending magnet looks like an ellipse with sharp edges, defined by scattering kinematics.
- Vertical size of this ellipse is independent on the beam energy. Low electron beam emittance (vertical) is required to observe the original cross section.
- Horizontal size of the ellipse depends on the product of two parameters, $\gamma\theta_0$, where $\gamma = \varepsilon_0/m$ and θ_0 is the beam bending angle in the same dipole. Thus $\gamma\theta_0$ can be measured with a relative statistical accuracy of at least 10^{-4}.
- Either transverse or longitudinal beam polarization can be evaluated from the same measurement.

Thank You!