Inverse Compton Scattering of Beamstrahlung Radiation on the Oncoming Bunch Particles at FCC-ee.

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August 28, 2017

Abstract

Eugene Levichev asked me how significant can be the effect of scattering of Beamstrahlung radiation on the oncoming bunch particles at FCC-ee. This note contains my basic assessments regarding this issue.

1 Radiation in the field of oncoming bunch

With the passage of ultrarelativistic electrons through the field of the oncoming bunch, average bending radii ρ_x, ρ_y and angles θ_x, θ_y of the electron were estimated in [1]:

$$\rho_x \simeq \rho_y \simeq \frac{\gamma\sigma_z\sigma_x}{N_p r_e}, \text{ where } N_p \text{ is the number of particles in the oncoming bunch,} \quad (1)$$

$$\theta_x \simeq \theta_y \simeq \frac{N_p r_e}{\gamma\sigma_x\sqrt{1+\phi^2}}, \ \phi \text{ is the Piwinski parameter: } \phi = \frac{\sigma_z}{\sigma_x}\mathrm{tg}\Theta, \quad (2)$$

$\sigma_x \gg \sigma_y$ is the transverse horizontal bunch size, σ_z is the longitudinal bunch size, Θ is the beams crossing angle (further $\Theta = 0.015$).

The energy emitted by an electron bent by angle θ with radius ρ is expressed as:

$$U = \frac{2}{3}mc^2\frac{r_e\theta}{\rho}\gamma^4, \quad (3)$$

where r_e and mc^2 are the classical electron radius and its rest energy, $\gamma = E/mc^2$. The critical energy of the synchrotron radiation photons and their energy spectrum [2] are described as:

$$E_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3, \quad (4)$$

$$\frac{dn}{dE_\gamma} = \frac{27}{8}\frac{U}{E_c^2}\left(\int_0^z \mathrm{Ai}(x)dx - \frac{1}{3} - 2\frac{\mathrm{Ai}'(z)}{z}\right), \text{ where } z = \left(\frac{3E_\gamma}{2E_c}\right)^{2/3}, \quad (5)$$

$\mathrm{Ai}(z)$ и $\mathrm{Ai}'(z)$ – are the Airy function and its derivation. The average number of photons emitted by each electron is expressed as $\langle n_\gamma\rangle = \frac{15\sqrt{3}}{8}(U/E_c)$.

Such photons could be scattered on the electrons (positrons) of the oncoming bunch. The probability of such a process is described by the Compton scattering cross section and the γe luminosity. The γe luminosity in our case should not be very different from the $e e$ luminosity cause the emitted photons are very collinear with the electrons. The Compton scattering cross section for the photon with energy E_γ and the head-on electron with energy E_0 [3] depends on the parameter $\kappa = 4E_\gamma E_0/(mc^2)^2$ only:

$$\sigma(\kappa) = \frac{2\pi r_e^2}{\kappa}\left(\left[1 - \frac{4}{\kappa} - \frac{8}{\kappa^2}\right]\log(1+\kappa) + \frac{1}{2}\left[1 - \frac{1}{(1+\kappa)^2}\right] + \frac{8}{\kappa}\right). \quad (6)$$

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2 Estimations for FCC-ee

Let us take the main design parameters of FCC-ee:

Table 1: FCC-ee parameters

Beam energy, E_0	[GeV]	45.6	80.0	120.0	175.0
Beam current	[mA]	1390	147	29	6
Bunches / ring		70760	7280	826	64
Particles / bunches		4.00e+10	4.10e+10	7.10e+10	2.04e+11
σ_x at I.P.	[m]	6.36e-06	1.67e-05	2.51e-05	3.66e-05
σ_z at I.P.	[m]	4.10e-03	2.30e-03	2.20e-03	2.90e-03
Piwinsky parameter		9.66	2.06	1.31	1.19
ρ_x, ρ_y	[m]	2.07e+01	5.22e+01	6.48e+01	6.32e+01
θ_x, θ_y	[rad]	2.04e-05	1.92e-05	2.05e-05	2.95e-05
U	[eV]	6.02e+04	2.13e+05	9.26e+05	6.16e+06
E_c	[eV]	1.02e+07	2.18e+07	5.91e+07	1.88e+08
$\langle n_\gamma \rangle$		0.019	0.032	0.051	0.106
Luminosity/IP	[cm^{-2}s^{-1}]	1.37e+36	1.64e+35	4.60e+34	1.40e+34

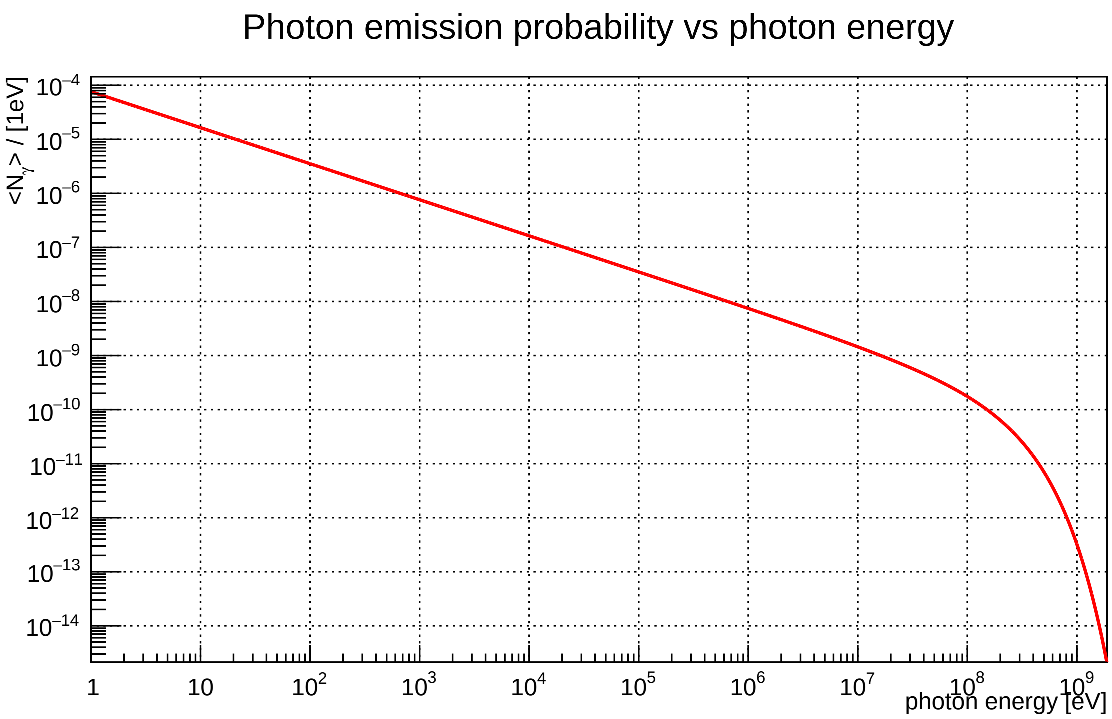

Figure 1: The photon emission probability vs its energy per electron in one collision. Parameters for calculation where taken from the last column of Table 1 ($E_0 = 175$ GeV).

The scattering probability for the photons with energy spectrum from Figure 1 depends solely on E_γ cause E_0 is fixed by FCC-ee operation energy. This is defined by equation (6):

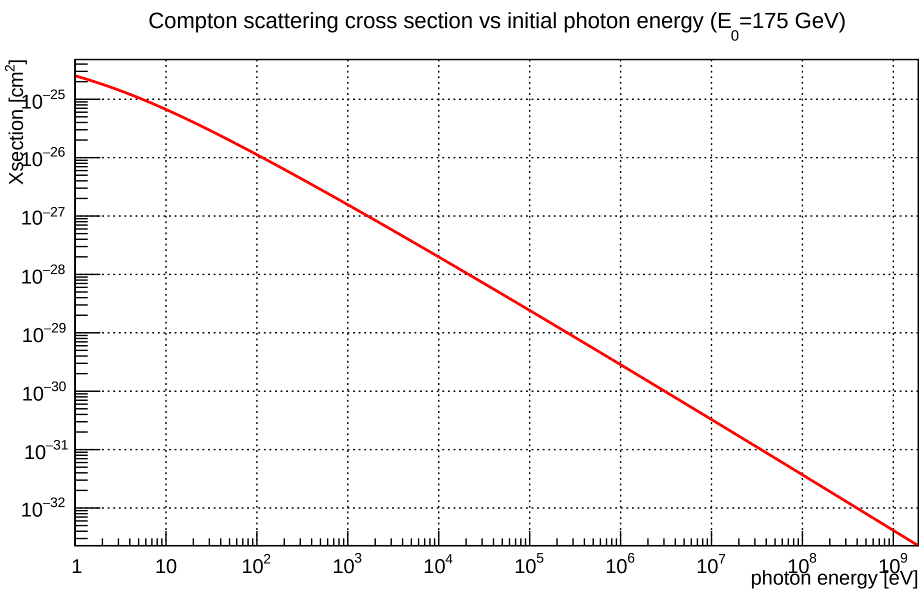

Figure 2: The Compton cross section vs E_γ.

The equivalent cross section for the process, when an electron emits a photon in the field of oncoming bunch, and then this photon is scattered on the positron of the oncoming bunch is defined by the product of equations (5) and (6). The result is shown in Figure 3:

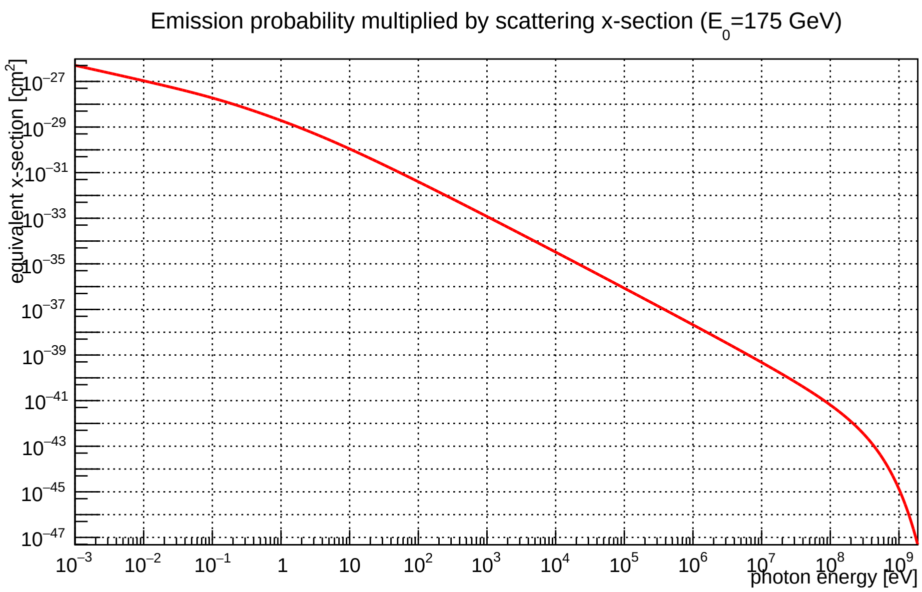

Figure 3: The product of emission probability and backscattering cross section vs E_γ.

3 Summary

The process of consequent radiation of relatively soft photon and then its backscattering on the oncoming electron will lead to production of hard photon radiation directed along with the beam axis. The energies of such photons will be the same order as the energies of initial electrons, so these electrons will be lost somewhere near IP. To estimate the magnitude of such an effect, we need to integrate our modified cross section (see Figure 3) and multiply the result by the luminosity. With the assumption that $L_{e\gamma} \simeq L_{ee}$ we have obtained the following results:

Table 2: Summary of results

Beam energy, E_0	[GeV]	45.6	80.0	120.0	175.0
Luminosity/IP, L	[cm^{-2}s^{-1}]	$1.37 \cdot 10^{36}$	$1.64 \cdot 10^{35}$	$4.6 \cdot 10^{34}$	$1.4 \cdot 10^{34}$
Number of bunches, N_b		70760	7280	826	64
Particles per bunch, N_e		$4.0 \cdot 10^{10}$	$4.1 \cdot 10^{10}$	$7.1 \cdot 10^{10}$	$2.0 \cdot 10^{11}$
equivalent cross section, σ^*	[$\times 10^{-28}$cm^2]	1.2	1.3	1.3	1.6
$L \cdot \sigma^*$	[1/s]	$1.6 \cdot 10^8$	$2.1 \cdot 10^7$	$6.0 \cdot 10^6$	$2.2 \cdot 10^6$
$L \cdot \sigma^* / N_b$	[1/s]	$2.3 \cdot 10^3$	$2.9 \cdot 10^3$	$7.2 \cdot 10^3$	$3.5 \cdot 10^4$
Beam lifetime impact τ^*	[s]	$1.7 \cdot 10^7$	$1.4 \cdot 10^7$	$0.9 \cdot 10^7$	$0.6 \cdot 10^7$

E. g. at $E_0 = 45.6$ GeV the equivalent cross section of the process under investigation is about 12 mb, that will produce about $1.6 \cdot 10^8$ high energy (tens of GeV) photons per second at nominal FCC-ee design parameters, taken from Table 1. However, the impact on the beam lifetime if very low, $\tau^* \simeq 1.7 \cdot 10^7$ s.

The estimations above are the subject for discussion, the results are preliminary and require verification.

References

[1] A. Bogomyagkov, E. Levichev, and D. Shatilov, "Beam-beam effects investigation and parameters optimization for a circular e+e- collider at very high energies," *Physical Review Special Topics - Accelerators and Beams*, vol. 17, no. 4, 2014.

[2] A. Hofmann, *The Physics of Synchrotron Radiation.* Cambridge University Press, 2004.

[3] V. Berestetskii, E. Lifshitz, and L. Pitaevskii, *Quantum Electrodynamics.* Course of theoretical physics, Butterworth-Heinemann, 1982.