

Function to describe photopeak shape ($\sigma_m = K_0\sigma_s$):

$$f(x) = \begin{cases} 0 < x - x_0 < +\infty : & \exp\left[-\frac{(x - x_0)^2}{2\sigma^2}\right] \\ -K_1\sigma_m < x - x_0 \leq 0 : & C + (1 - C) \cdot \exp\left[-\frac{(x - x_0)^2}{2\sigma_m^2}\right] \\ -\infty < x - x_0 \leq -K_1\sigma_m : & C + (1 - C) \cdot \exp\left[K_1\left(\frac{x - x_0}{\sigma_m} + \frac{K_1}{2}\right)\right] \end{cases} \quad (1)$$

Convolution with gaussian ($x - x_0 \rightarrow x$):

$$S(x) = \int dy \begin{cases} [0 \div +\infty] : & \exp\left[-\frac{y^2}{2\sigma^2}\right] \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] + \\ [-K_1\sigma_m \div 0] : & + \left(C + (1 - C) \cdot \exp\left[-\frac{y^2}{2\sigma_m^2}\right]\right) \cdot \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] + \\ [-\infty \div -K_1\sigma_m] : & + \left(C + (1 - C) \cdot \exp\left[K_1\left(\frac{y}{\sigma_m} + \frac{K_1}{2}\right)\right]\right) \cdot \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] \end{cases} \quad (2)$$

The same, with $D \equiv 1 - C$:

$$S(x) = \int dy \begin{cases} [0 \div +\infty] : & \exp\left[-\frac{y^2}{2\sigma^2}\right] \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] + \\ [-\infty \div 0] : & + C \cdot \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] + \\ [-K_1\sigma_m \div 0] : & + D \cdot \exp\left[-\frac{y^2}{2\sigma_m^2}\right] \cdot \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] + \\ [-\infty \div -K_1\sigma_m] : & + D \cdot \exp\left[K_1\left(\frac{y}{\sigma_m} + \frac{K_1}{2}\right)\right] \cdot \exp\left[-\frac{(x - y)^2}{2\sigma_s^2}\right] \end{cases} \quad (3)$$

Rewrite 1:

$$S(x) = \exp\left(-\frac{x^2}{2\sigma_s^2}\right) \int dy \begin{cases} [0 \div +\infty] : & \exp\left[-\frac{y^2}{2\sigma^2}\right] \exp\left[\frac{2xy - y^2}{2\sigma_s^2}\right] + \\ [-\infty \div 0] : & + C \cdot \exp\left[\frac{2xy - y^2}{2\sigma_s^2}\right] + \\ [-K_1\sigma_m \div 0] : & + D \cdot \exp\left[-\frac{y^2}{2\sigma_m^2}\right] \cdot \exp\left[\frac{2xy - y^2}{2\sigma_s^2}\right] + \\ [-\infty \div -K_1\sigma_m] : & + D \cdot \exp\left[K_1\left(\frac{y}{\sigma_m} + \frac{K_1}{2}\right)\right] \cdot \exp\left[\frac{2xy - y^2}{2\sigma_s^2}\right] \end{cases} \quad (4)$$

Rewrite 2 ($q \equiv 1/2\sigma^2$, $q_s \equiv 1/2\sigma_s^2$, $q_m \equiv 1/2\sigma_m^2$):

$$S(x) = \exp(-q_s x^2) \int dy \begin{cases} [0 \div +\infty] : & \exp(-y^2(q + q_s) + 2yxq_s) + \\ [-\infty \div 0] : & + C \cdot \exp(-y^2 q_s + 2yxq_s) + \\ [-K_1\sigma_m \div 0] : & + D \cdot \exp(-y^2(q_m + q_s) + 2yxq_s) + \\ [-\infty \div -K_1\sigma_m] : & + D \cdot \exp(-y^2 q_s + y(2xq_s + K_1\sqrt{2q_m})) \exp(K_1^2/2) \end{cases} \quad (5)$$

From WolframAlfa:

$$\int_{L_1}^{L_2} \exp(-ax^2 + bx) dx = -\frac{1}{2} \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right) \left(\operatorname{erf}\left(\frac{2aL_1 - b}{2\sqrt{a}}\right) + \operatorname{erf}\left(\frac{b - 2aL_2}{2\sqrt{a}}\right)\right) \quad (6)$$

$$\int_0^{+\infty} \exp(-ax^2 + bx) dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right) \operatorname{erfc}\left(\frac{-b}{2\sqrt{a}}\right) \quad (7)$$

$$\int_{-\infty}^0 \exp(-ax^2 + bx) dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right) \operatorname{erfc}\left(\frac{b}{2\sqrt{a}}\right) \quad (8)$$

$$\int_{-L_1}^0 \exp(-ax^2 + bx) dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right) \left(\operatorname{erfc}\left(\frac{b}{2\sqrt{a}}\right) - \operatorname{erfc}\left(\frac{2aL_1 + b}{2\sqrt{a}}\right)\right) \quad (9)$$

$$\int_{-\infty}^{-L_1} \exp(-ax^2 + bx) dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right) \operatorname{erfc}\left(\frac{2aL_1 + b}{2\sqrt{a}}\right) \quad (10)$$

Hence:

$$\begin{aligned}
S(x) &= \frac{\sqrt{\pi}}{2} \exp(-q_s x^2) \times \\
&\times \left\{ \frac{1}{\sqrt{q+q_s}} \exp\left(\frac{x^2 q_s^2}{(q+q_s)}\right) \operatorname{erfc}\left(\frac{-x q_s}{\sqrt{q+q_s}}\right) + \frac{C}{\sqrt{q_s}} \exp(x^2 q_s) \operatorname{erfc}(x\sqrt{q_s}) + \right. \\
&+ \frac{D}{\sqrt{q_m+q_s}} \exp\left(\frac{x^2 q_s^2}{q_m+q_s}\right) \left(\operatorname{erfc}\left(\frac{x q_s}{\sqrt{q_m+q_s}}\right) - \operatorname{erfc}\left(\frac{(q_m+q_s)K_1\sigma_m+xq_s}{\sqrt{q_m+q_s}}\right) \right) + \\
&\left. + \frac{D}{\sqrt{q_s}} \exp\left(\frac{K_1^2}{2}\right) \exp\left(\frac{(2xq_s+K_1\sqrt{2q_m})^2}{4q_s}\right) \operatorname{erfc}\left(\frac{2q_s K_1\sigma_m+2xq_s+K_1\sqrt{2q_m}}{2\sqrt{q_s}}\right) \right\}
\end{aligned} \tag{11}$$

Again:

$$\begin{aligned}
S(x) &= \frac{\sqrt{\pi}}{2} \exp(-q_s x^2) \times \left\{ \frac{1}{\sqrt{q+q_s}} \exp\left(\frac{x^2 q_s^2}{(q+q_s)}\right) \operatorname{erfc}\left(\frac{-x q_s}{\sqrt{q+q_s}}\right) + \frac{C}{\sqrt{q_s}} \exp(x^2 q_s) \operatorname{erfc}(x\sqrt{q_s}) + \right. \\
&+ \frac{D}{\sqrt{q_m+q_s}} \exp\left(\frac{x^2 q_s^2}{q_m+q_s}\right) \left(\operatorname{erfc}\left(\frac{x q_s}{\sqrt{q_m+q_s}}\right) - \operatorname{erfc}\left(K_1\sigma_m\sqrt{q_m+q_s} + \frac{xq_s}{\sqrt{q_m+q_s}}\right) \right) + \\
&\left. + \frac{D}{\sqrt{q_s}} \exp\left(\frac{K_1^2}{2}\left(1 + \frac{q_m}{q_s}\right)\right) \exp(x^2 q_s) \exp(xK_1\sqrt{2q_m}) \operatorname{erfc}\left(K_1\sigma_m\sqrt{q_s} + x\sqrt{q_s} + \frac{K_1\sqrt{q_m}}{\sqrt{2q_s}}\right) \right\}
\end{aligned} \tag{12}$$

Back from q 's to σ 's:

$$\begin{aligned}
S(x) &= \frac{\sqrt{\pi}}{2} \exp\left(-\frac{x^2}{2\sigma_s^2}\right) \times \left\{ \sqrt{\frac{2\sigma^2\sigma_s^2}{\sigma^2+\sigma_s^2}} \exp\left(\frac{x^2\sigma^2}{2\sigma_s^2(\sigma^2+\sigma_s^2)}\right) \operatorname{erfc}\left(\frac{-x\sigma}{\sigma_s\sqrt{2(\sigma^2+\sigma_s^2)}}\right) + \right. \\
&+ \sqrt{2}C\sigma_s \exp\left(\frac{x^2}{2\sigma_s^2}\right) \operatorname{erfc}\left(\frac{x}{\sqrt{2}\sigma_s}\right) + \\
&+ D\sqrt{\frac{2\sigma_m^2\sigma_s^2}{\sigma_m^2+\sigma_s^2}} \exp\left(\frac{x^2\sigma_m^2}{2\sigma_s^2(\sigma_m^2+\sigma_s^2)}\right) \left(\operatorname{erfc}\left(\frac{x\sigma_m}{\sigma_s\sqrt{2(\sigma_m^2+\sigma_s^2)}}\right) - \operatorname{erfc}\left(\frac{K_1\sqrt{\sigma_m^2+\sigma_s^2}}{\sqrt{2}\sigma_s} + \frac{x\sigma_m}{\sigma_s\sqrt{2(\sigma_m^2+\sigma_s^2)}}\right) \right) + \\
&\left. + \sqrt{2}D\sigma_s \exp\left(\frac{K_1^2}{2}\left(1 + \frac{\sigma_s^2}{\sigma_m^2}\right)\right) \exp\left(\frac{x^2}{2\sigma_s^2}\right) \exp\left(\frac{xK_1}{\sigma_m}\right) \operatorname{erfc}\left(\frac{K_1\sigma_m}{\sqrt{2}\sigma_s} + \frac{x}{\sqrt{2}\sigma_s} + \frac{K_1\sigma_s}{\sqrt{2}\sigma_m}\right) \right\}
\end{aligned} \tag{13}$$

Rewrite:

$$\begin{aligned}
S(x) &= \sqrt{\frac{\pi}{2}}\sigma_s \times \left\{ \frac{\sigma}{\sqrt{\sigma^2+\sigma_s^2}} \exp\left(-\frac{x^2}{2(\sigma^2+\sigma_s^2)}\right) \operatorname{erfc}\left(\frac{-x\sigma}{\sigma_s\sqrt{2(\sigma^2+\sigma_s^2)}}\right) + C \cdot \operatorname{erfc}\left(\frac{x}{\sqrt{2}\sigma_s}\right) + \right. \\
&+ D \cdot \frac{\sigma_m}{\sqrt{\sigma_m^2+\sigma_s^2}} \exp\left(-\frac{x^2}{2(\sigma_m^2+\sigma_s^2)}\right) \left(\operatorname{erfc}\left(\frac{x\sigma_m}{\sigma_s\sqrt{2(\sigma_m^2+\sigma_s^2)}}\right) - \operatorname{erfc}\left(\frac{K_1(\sigma_m^2+\sigma_s^2)+x\sigma_m}{\sigma_s\sqrt{2(\sigma_m^2+\sigma_s^2)}}\right) \right) + \\
&\left. + D \cdot \exp\left(\frac{K_1^2}{2}\left(1 + \frac{\sigma_s^2}{\sigma_m^2}\right) + K_1\frac{x}{\sigma_m}\right) \operatorname{erfc}\left(\frac{K_1(\sigma_s^2+\sigma_m^2)+x\sigma_m}{\sqrt{2}\sigma_s\sigma_m}\right) \right\}
\end{aligned} \tag{14}$$

Rewrite, devide to $2\pi\sigma\sigma_s$, $\sigma_m = K_0\sigma$:

$$\begin{aligned}
S(x) &= \frac{1}{2\sqrt{2\pi}} \left\{ \frac{1}{\sqrt{\sigma^2+\sigma_s^2}} \exp\left(\frac{-x^2}{2(\sigma^2+\sigma_s^2)}\right) \operatorname{erfc}\left(\frac{-x\sigma}{\sigma_s\sqrt{2(\sigma^2+\sigma_s^2)}}\right) + \frac{C}{\sigma} \operatorname{erfc}\left(\frac{x}{\sqrt{2}\sigma_s}\right) + \right. \\
&+ \frac{DK_0}{\sqrt{\sigma_m^2+\sigma_s^2}} \exp\left(\frac{-x^2}{2(\sigma_m^2+\sigma_s^2)}\right) \left(\operatorname{erfc}\left(\frac{x\sigma_m}{\sigma_s\sqrt{2(\sigma_m^2+\sigma_s^2)}}\right) - \operatorname{erfc}\left(\frac{K_1(\sigma_m^2+\sigma_s^2)+x\sigma_m}{\sigma_s\sqrt{2(\sigma_m^2+\sigma_s^2)}}\right) \right) + \\
&\left. + \frac{D}{\sigma} \exp\left(\frac{K_1^2}{2}\left(1 + \frac{\sigma_s^2}{\sigma_m^2}\right) + K_1\frac{x}{\sigma_m}\right) \operatorname{erfc}\left(\frac{K_1(\sigma_m^2+\sigma_s^2)+x\sigma_m}{\sqrt{2}\sigma_s\sigma_m}\right) \right\}
\end{aligned} \tag{15}$$

Check with $C = 0$, $D = 1$, $\sigma_m = \sigma$, $K_0 = 1$, $K_1 = \xi$:

$$\begin{aligned}
S(x) &= \frac{1}{2\sqrt{2\pi}} \left\{ \frac{1}{\sqrt{\sigma^2+\sigma_s^2}} \exp\left(\frac{-x^2}{2(\sigma^2+\sigma_s^2)}\right) \operatorname{erfc}\left(-\frac{\xi(\sigma^2+\sigma_s^2)+x\sigma}{\sigma_s\sqrt{2(\sigma^2+\sigma_s^2)}}\right) + \right. \\
&\left. + \frac{1}{\sigma} \exp\left(\frac{\xi^2}{2}\left(1 + \frac{\sigma_s^2}{\sigma^2}\right) + \xi\frac{x}{\sigma}\right) \operatorname{erfc}\left(\frac{\xi(\sigma_s^2+\sigma^2)+x\sigma}{\sqrt{2}\sigma\sigma_s}\right) \right\}
\end{aligned} \tag{16}$$